https://doi.org/10.15407/scine21.02.093

ROMANIUK, O. D. (https://orsid.org/000-0003-1931-5507),
CHASOV, D. P. (https://orsid.org/0000-0003-3830-693X),
ROMANIUK, Ya. O. (https://orsid.org/0009-0001-1343-2211),
SASOV, O. O. (https://orsid.org/0000-0002-8697-6324),
and BEIHUL, V. O. (https://orsid.org/0000-0003-1350-0656)

Dniprovsky State Technical University,
2, Dniprobudivska St., Kamianske, 51918, Dnipropetrovska Oblast, Ukraine,
+380 569 56 0665, science@dstu.dp.ua

OPTIMIZATION OF ROTATING ASSEMBLY MASSES IN MECHANICAL TRANSMISSIONS OF MACHINE UNITS

Introduction. *The stabilization of kinematic and dynamic characteristics in machine assemblies, which exhibit periodic changes during steady-state operation, has necessitated the development of methods for optimizing flywheel masses to enhance the productivity of machine executive bodies.*

Problem Statement. *Stabilizing the parameters of machine assemblies with additional flywheel masses increases the overall weight and inertia of the mechanism. Therefore, it has become essential to design mechanical transmissions with optimized rotating assembly masses that can perform the function of flywheel masses.*

Purpose. *This study has aimed to develop a method for estimating the mass of a rotary assembly in a mechanical transmission during the preliminary design stage, utilizing the power parameters of the machine unit.*

Materials and Methods. *Analytical approaches have been employed to study the dependencies in a machine unit model and its rotary assembly, featuring cylindrical gears.*

Results. *A functional dependence of the rotary assembly mass on torque, gear ratio, and the mechanical characteristics of gear wheel and shaft materials has been established. Mass coefficients for gears and shafts have been derived, allowing variation through material selection and heat treatment. This expands the range of optimization options and simplifies solving multivariate design problems.*

Conclusions. *The proposed method for estimating the rotary assembly mass at the preliminary design stage, based on the machine unit's power parameters, has shown promise. It provides a clear framework for aligning the masses of assembly components with the required flywheel moment, ensuring stabilization of the machine unit's kinematic and dynamic characteristics.*

Keywords: mechanical transmission, machine unit, flywheel, assembly, mass, gear wheel, shaft, torque.

Citation: Romaniuk, O. D., Chasov, D. P., Romaniuk, Ya. O., Sasov, O. O., and Beihul, V. O. (2025). Optimization of Rotating Assembly Masses in Mechanical Transmissions of Machine Units. *Sci. innov.*, 21(2), 93—100. https://doi.org/10.15407/scine21.02.093

At the present stage, the most effective approach to addressing optimization problems in mechanics involves various modeling methods. These methods provide a comprehensive understanding of the interconnections between the elements of the designed machine or mechanism and enable targeted modifications of parameters or, when necessary, the overall structure [1—4]. Modeling data allows the evaluation of a technical system's behavior under both ideal and extreme operating conditions.

The modeling method has enabled the variation of mechanism parameters such as the size and mass of components and assemblies, the gear ratio of mechanical transmissions, and kinematic and force parameters. This approach facilitates the analysis of their influence on the mechanism's performance or its individual components. Such a methodology provides insights into the periodic changes in the acting forces and the transfer function of a mechanical transmission, which in turn determine the corresponding variations in the kinematic and dynamic characteristics of the machine unit's drive in a steady-state operating mode.

Typically, the task of reducing the amplitude of fluctuations in the kinematic and dynamic characteristics of the machine unit's drive has been addressed by installing a flywheel, which increases the equivalent moment of inertia [5—9]. While the use of a flywheel significantly reduces the amplitude of angular velocity fluctuations in the driving link, it also substantially increases the mechanism's weight, leading to higher inertia.

To address the limitations of this approach, it is advisable to design new mechanical transmissions during the preliminary configuration stage of the machine unit's drive, with rotating assembly masses capable of fulfilling the flywheel's function.

The primary elements of mechanical transmissions are assemblies that perform rotational motion. In some cases, the mass of such assemblies constitutes more than two-thirds of the gearbox's weight. This factor can be leveraged to eliminate the disadvantages of additional flywheel masses by designing new mechanisms with the moments of inertia of rotating masses optimized to function as a flywheel. Furthermore, designing suitable transmissions is necessary because finding a standard gearbox with the required parameters is practically impossible.

This approach enables the adjustment of the equivalent moment of inertia without altering the parameters of the machine's executive body. In this case, the role of the flywheel is performed by the rotating assembly masses of the mechanical transmission.

Thus, solving the stated problem involves selecting the mass of the rotating assembly of the mechanical transmission during the preliminary design stage to ensure it corresponds to the flywheel moment of inertia.

The unique aspect of designing a mechanical transmission assembly with optimized mass lies in the significantly greater number of unknown design variables compared to the equations of constraints. Therefore, the task involves reducing the number of unknowns in the objective function, imposing certain restrictions on the range of their variation [10, 11], and determining the impact of relevant parameters on the nature of mass variation. Consequently, it is advisable to consider optimizing the mass of individual components of the assembly and imposing constraints on certain parameters of these components.

The corresponding problem has been addressed in studies [12—14], but only with a focus on optimizing the mass of the gear wheels in mechanical transmissions. Considering the multi-criteria and multi-variant nature of solving optimization problems for machine unit parameters and their components, these studies have derived dependencies for determining the mass of gear wheels in the following form:

$$m_1 = A(1+u)z_1^3;$$

$$m_2 = A\left(1+\frac{1}{u}\right)z_2^3,$$

where: m_1 is the mass of the pinion, kg; m_2 is the mass of the wheel, kg; u is the gear ratio of the mechanical transmission; z_1 is the number of teeth on the pinion; z_2 is the number of teeth on the wheel; A is the mass coefficient of the gear wheels, kg.

Recommendations for selecting the mass coefficient A, considering the interdependent geometric parameters of gear transmissions, have been presented in studies [13, 14]. These recommendations are based on reducing the number of unknown design parameters and introducing specific constraints on the intervals of their variation.

A limitation of the equations obtained for estimating the mass of gear wheels during the preliminary design stage is the mass coefficient A that is determined by the following dependence:

$$A = \frac{1}{8}\pi\rho\psi_{ba}\left(\frac{m_n}{\cos\beta}\right)^3,$$

where: ψ_{ba} is the coefficient of the gear rim width to center distance; m_n is the normal module of meshing, mm; β is the helix angle, degree; ρ is the specific gravity of the gear material, kg/m^3.

Thus, the mass coefficient A is a function of three variables representing the geometric parameters, $A = f(\psi_{ba}, m_n, \beta)$ under the condition that $\rho = const$. These parameters that are not definitively determined during the preliminary design stage, significantly complicate the process of selecting the optimal mass for the gear wheels.

Using a similar approach, studies [15, 16] have derived the formula for calculating the mass of a shaft model element with bearings, at the predesign stage:

$$m_b = \frac{1}{4}\pi\rho K d_{b_1}^3,$$

where: m_b is the mass of the shaft with inner bearing rings, kg; Kis the general structural coefficient for recalculating the geometric parameters of the shaft model elements with bearings; d_{b_1} is diameter of the initial section of the shaft, mm; ρ is the specific gravity of the shaft material, kg/m^3.

Thus, the mass of the rotary assembly of a mechanical transmission at the predesign stage, according to studies [12—16], is calculated by the following formula:

$$m = m_{1(2)} + m_b.$$

Considering the interdependent relationships between the geometric parameters of gear transmissions, it is more practical to develop a methodology for estimating the mass of the rotary assembly of a mechanical transmission at the predesign stage based on the power parameters, which are the primary characteristics of the machine unit.

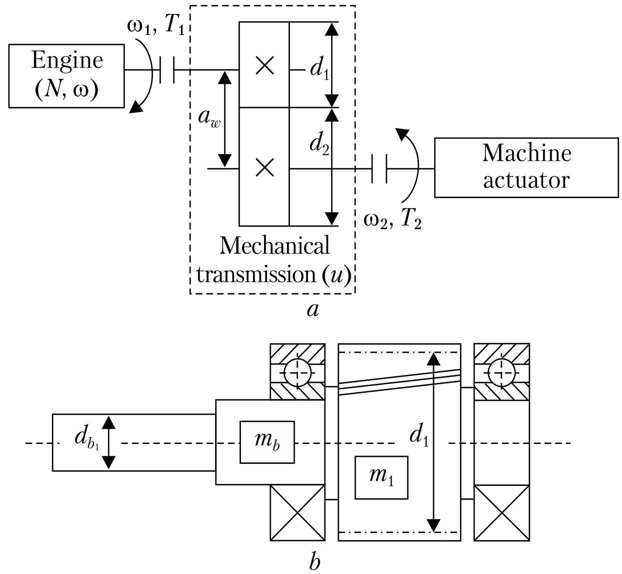

Fig. 1. Models of the machine unit (a) and the mechanical transmission assembly (b)

The models of the machine unit and the rotary assembly of the mechanical transmission are shown in Fig. 1.

The key model parameters are as follows: N is the engine power, W; ω is the angular velocity of the engine shaft, 1/s ; ω_1 is the angular velocity of the high-speed shaft, 1/s; ω_2 is the angular velocity of the low-speed shaft, 1/s; T_1, T_2 are the torque on the high-speed and the low-speed shafts, respectively, $N{\cdot}m$; d_1, d_2 are the pitch diameters of the pinion and the wheel, respectively, mm; a_w is the center distance, mm; d_{b1} is the diameter of the initial section of the shaft, mm.

The geometric characteristics of the gear wheels in a mechanical transmission are functions of the center distance: $m_n = f(a_w)$; $\psi_{ba} = f(a_w)$; $\beta = f(a_w)$; $z_i = f(a_w)$, which is function of the torque and the gear ratio, $a_w = f(T, u)$. The geometric parameters of the shaft are also functions of the torque: $K = f(d_{b_1})$; $d_{b_1} = f(T)$.

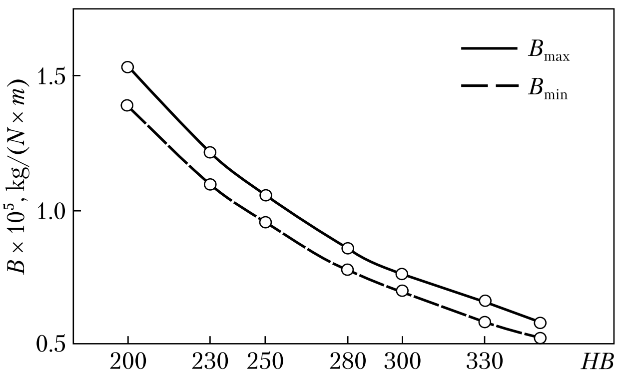

Fig. 2. Dependance of the mass coefficient B on the teeth's working surface hardness HB

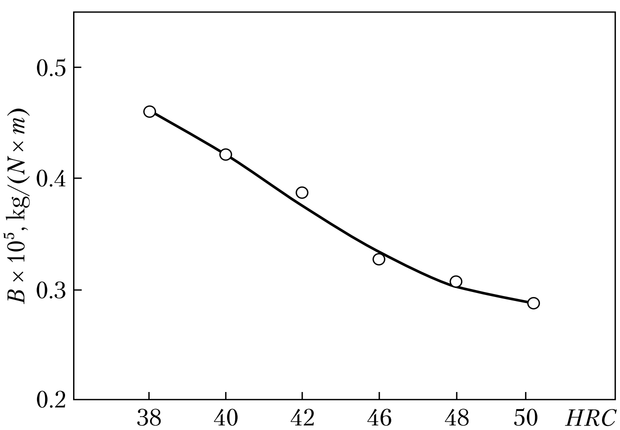

Fig. 3. Dependance of the mass coefficient B on the teeth's working surface hardness HRC

Thus, the feasibility of optimizing the geometric parameters of the rotary assembly of the mechanical transmission, which determine its mass based on the power characteristics of the machine unit, is mathematically substantiated, as also confirmed by the studies presented in [17].

The mass of the wheel is estimated by the relationship provided in [13]:

$$m_2 = u^2 m_1,$$

Therefore, further research is made only for the pinion.

According to [12], the mass of the pinion is determined by the formula:

$$m_1 = \frac{1}{8}\pi\rho\psi_{ba}\left(\frac{m_n}{\cos\beta}\right)^3(1+u)z_1^3.$$

Given the pitch diameter of the pinion:

$$d_1 = \frac{m_n z_1}{\cos\beta},$$

the mass is determined by the equation:

$$m_1 = \frac{1}{8}\pi\rho\psi_{ba}(1+u)d_1^3.$$

Given that the center distance is

$$a_w = \frac{1}{2}(d_1 + d_2) = \frac{1}{2}d_1(1+u),$$

let us define the pitch diameter of the pinion as function of

$$d_1 = \frac{2a_w}{1+u}.$$

Having substituted into the given equation

$$a_w = K_a(u+1)\sqrt[3]{\frac{E_{np}T_2K_{H\beta}}{[\sigma_H]^2u^2\psi_{ba}}}.$$

Given that $T_2 = T_{1u}$, we obtain:

$$d_1 = 2K_a\sqrt[3]{\frac{E_{np}T_2K_{H\beta}}{[\sigma_H]^2u^2\psi_{ba}}},$$

where: E_{np} is the equivalent modulus of elasticity of the gear wheel materials, MPa; $[\sigma_H]$ is the permissible contact stresses, MPa; $K_{H\beta}$ is the coefficient accounting for the uneven load distribution across the gear wheel face width; K_a is the coefficient accounting for the helix angle of the gear teeth.

The equivalent modulus of elasticity of the gear wheel materials is determined by the formula:

$$E_{np} = \frac{2E_1E_2}{E_1 + E_2},$$

where E_1, E_2 are the moduli of elasticity of the pinion and the wheel material, MPa.

Accordingly, the equation for the mass of the pinion is written as follows:

$$m_1 = BT_1\left(1 + \frac{1}{u}\right),$$

where: $B = \pi\rho K_a^3 K_{H\beta}\frac{E_{np}}{[\sigma_H]^2}$ is the mass coefficient that accounts for the properties of the gear material, kg/$(N \times m)$.

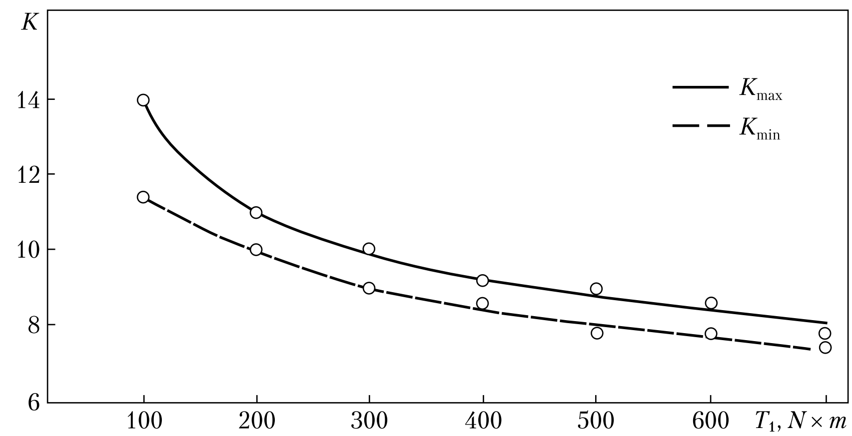

Fig. 4. Dependences of the coefficient K on T_1

The derived dependence enables a more accurate determination of the gear's mass at the preliminary design stage, as the parameters T_1 and u are unambiguously known, and the equation excludes the parameter z_1 that is also not clearly defined. The mass coefficient B, compared to the mass coefficient A, offers significant advantages since the mechanical properties of the gear material and the helix angle of the tooth are definitively known at the predesign stage.

Moreover, the coefficient B allows for variation by selecting the gear material or applying heat treatment, significantly broadening the range for finding an optimal solution and simplifying the task. The coefficients $K_{H\beta}$ and K_a are chosen following the recommendations for gear calculations [18]. It is worth noting that the helix angle of the tooth has minimal impact on the numerical values of the coefficient B.

According to [15], the dependence for estimating the mass of a shaft element with bearings at the predesign stage

$$m_b = \frac{1}{4}\pi\rho K d_{b_1}^3,$$

given that

$$d_{b_1} = \sqrt[3]{\frac{T_1}{0.2[\tau]}}$$

is written as follows:

$$m_b = \left(\frac{5\pi\rho}{4[\tau]}\right) K T_1 = XKT_1,$$

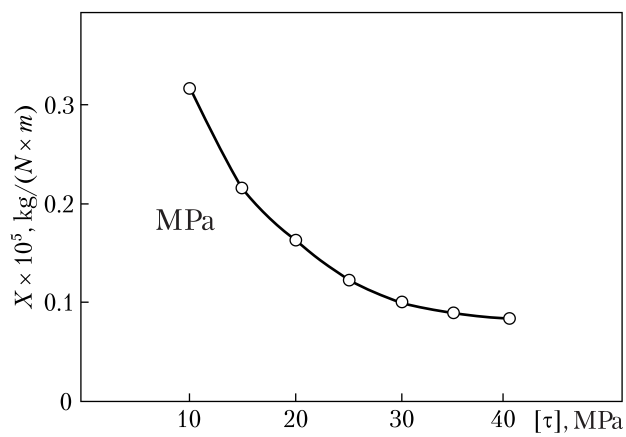

Fig. 5. Dependance of the shaft mass X on [τ]

where: $X = 3.927\ \rho/[\tau]$ is the mass coefficient that takes into account the material properties of the shaft., kg/(N × m); [τ] is the permissible tangential stress of the shaft material, MPa.

Accordingly, the mass of the rotating assembly of the mechanical transmission at the predesign stage can be estimated by the formula:

$$m = m_1 + m_b = \left\{ B\left(1 + \frac{1}{u}\right) + XK \right\} T_1.$$

According to this, for the preliminary estimation of the mass of the rotating assembly, the following dependencies are required: $T_1 = f(N, \omega)$; $B = f(HB)$ and $B = f(HRC)$; $K = f(T_1)$; $X = f([\tau])$.

The mass coefficient B is calculated for carbon and alloy steels for heat treatments such as normalization, improvement, and volumetric hardening. For carbon steels of grades 20, 30, 35, 40, 45, the modulus of elasticity is $E = 1.96 / 2.06 \times 10^5$ MPa, and the specific gravity is $\rho = 7850$ kg/m^3 [19]. For alloy steels of grades 30X, 45X, 40XN, 45XN, the modulus of elasticity is $E = 2.06 / 2.16 \times 10^5$ MPa, and the specific gravity is $\rho = 7820$ kg/m^3 [19].

The permissible contact stress is determined by the formula:

$$[\sigma_H] = \frac{\sigma_{H\lim} \cdot K_{HL}}{[n_H]},$$

where: $\sigma_{H\lim}$ is the contact fatigue limit of the material of gear wheels, MPa; K_{HL} is the durability coefficient; and $[n_H]$ is the safety factor coefficient.

According to [18], when the heat treatment involves normalization or improvement (HB180—350), the contact fatigue limit is $\sigma_{H\lim} = 2HB + 70$, while for heat treatment with hardening (HRC38—50), the contact fatigue limit is $\sigma_{H\lim} = 18HRC + 150$. Accordingly, the coefficients are $K_{HL} = 1$ and $[n_H] = 1.15$.

The dependence of the mass coefficient B on the hardness of the working surface of the teeth for carbon and alloy steels during heat treatment (normalization or improvement (HB180—350) is shown in Fig. 2.

The coefficient B_{max} has been calculated for alloy steels 30X, 45X, 40XH, and 45XH. The elasticity modulus is $E = 2{,}16 \times 10^5$ MPa. The specific gravity is $\rho = 7820$ kg/m^3. The coefficient B_{min} has been calculated for carbon steels 20, 30, 35, 40, and 45. The elasticity modulus is $E = 1{,}96 \times 10^5$ MPa. The specific gravity is $\rho = 7850$ kg/m^3.

The dependence of the mass coefficient B on the hardness of the working surface of gear teeth for carbon and alloy steels subjected to volumetric hardening (HRC38—50) is shown in Fig. 3. Based on the analysis of the curves in Fig. 2, the mass coefficient B for volumetric hardening has been calculated for the averaged modulus of elasticity. $E = 2{,}06 \times 10^5$ MPa and the specific gravity $\rho = 7820$ kg/m^3.

The dependences of the coefficient K on the torque T_1 are presented in Fig. 4. The maximum and minimum values of the coefficient K have been calculated for the ranges of the geometric parameters of the shaft elements [18].

The variation in the coefficient X for carbon and alloy steels as a function of the permissible shear stress $[\tau]$ of the shaft material is shown in Fig. 5. Given that the gear is typically manufactured integrally with the shaft, the coefficient X has calculated for steels 30X, 45X, 40XH, and 45XH. The specific gravity is $\rho = 7820$ kg/m^3.

CONCLUSIONS

1. A functional dependency of the mass of the rotational assembly of a mechanical transmission on torque, gear ratio, and introduced mass coefficients that are functions of the mechanical properties of the gear and shaft materials, has been derived.

2. The introduced mass coefficients for gears and shafts allow for variation in material and heat treatment, significantly expanding the range of optimal design options and simplifying the solution to multi-criteria design problems.

3. It has been established that at the preliminary stage of estimating the mass of the rotational assembly of a mechanical transmission, averaged values of the coefficients B and K can be used, as the error does not exceed permissible deviations. Additionally, it was found that the helix angle of the gear teeth has a negligible impact on the numerical values of coefficient B.

4. The proposed method for estimating the mass of the rotational assembly of a mechanical transmission, whose moment of inertia corresponds to the flywheel moment of a flywheel, enables solving the problem of stabilizing the kinematic and dynamic characteristics of the machine assembly, as the corresponding force characteristics and mechanical properties of the gear and shaft materials are definitively determined during the predesign stage.

REFERENCES

1. Li, M., Azarm, S., Boyars, A. (2006). A new deterministic approach using sensitivity region measures for multi-objective robust and feasibility robust design optimization. *J. Mech. Des.*, 128(4), 874—883. https://doi.org/10.1115/1.2202884
2. Chasov, D. (2016). Determining the equation of surface of additional blade of a screw conveyor. *Eastern-European Journal of Enterprise Technologies (Engineering technological systems),* 5/1(83), 4—9. https://doi.org/10.15587/1729-4061.2016.80606
3. Chasov, D., Sorokina, L., Havrylin, S. (2017). Aspects of distance learning fo engineering sciences. *Effective Development of Teachers Skills in the Area of ICT and E-learning*, 319—331.
4. Beihul, O., Grischenko, D., Beihul, V., Chasov, D., Lepetova, A., Kolyada, B. (2020). Devising a procedure for calculating the designed strength of a kingpin-type load-carrying system for an articulated tractor container carrier. *Eastern-European Journal of Enterprise Technologies (Engineering technological systems)*, 5/7 (107), 22—29. https://doi.org/10.15587/1729-4061.2020.214133
5. Genta, G. (2014). *Kinetic Energy Storage: Theory and Practice of Advanced Flywheel Systems*. Butterworth-Heinemann.
6. Rupp, A., Baier, H., Mertiny, P., Secanell, M. (2016). Analysis of a flywheel energy storage system for light rail transit. *Energy,* 107, 625—638. https://doi.org/10.1016/j.energy.2016.04.051
7. Kim, S. J., Hayat, K., Nasir, S. U., Ha, S. K. (2014). Design and fabrication of hybrid composite hubs for a multi-rim flywheel energy storage system. *Composite Structures*, 107, 19—29. https://doi.org/10.1016/j.compstruct.2013.07.032
8. Romaniuk, O. D., Telipko, L. P. (2019). Optimization of the transmission function of the mechanical transmission in order to improve the accuracy of the machine unit. *Collection of scholarly papers of Dniprovsk State Technical University (Technical Sciences)*, 1(34), 34—38 [in Ukrainian]. https://doi.org/10.31319/2519-2884.34.2019.6
9. Romaniuk, O. D., Telipko, L. P. (2020). General methods of optimization of kinematic and dynamic parameters of the machine unit. *Collection of scholarly papers of Dniprovsk State Technical University (Technical Sciences)*, 2(37), 17—21. https://doi.org/10.31319/2519-2884.37.2020.4 [in Ukrainian].
10. Vasilieva, E., Kuzio, I. (2014). Optimization of the structural components of gearwheels of cylindrical reducing gears. *Econtechmod. An international quarterly journal on economics in technology, new technologies and modeling processes*, 1, 127—133.
11. Gulida, E. M., Vasylieva, O. E. (2007). Optimization of structural elements of cylindrical gears in the process of their design. *Problems of quality and durability of gears of reducers, their parts and assemblies,* 21, 191—197 [in Ukrainian].
12. Romaniuk, O. D. (2015). The choice of wing parts by weight of gear mechanisms based on optimizing the transfer function. *Mathematical Modeling*, 2(33), 52—54 [in Ukrainian].
13. Romaniuk, O. D. (2016). Optimization of the masses of gears mechanical transmission during the preliminary design. *Mathematical Modeling*, 2(35), 35—38 [in Ukrainian].
14. Romaniuk, O.D. (2018). Basic approaches to mass optimization of toothed wheels of mechanical transmission. *Mathematical Modeling*, 1(38), 118—123. https://doi.org/10.31319/2519-8106.1(38)2018.129038 [in Ukrainian].
15. Romaniuk, O. D. (2019). Development of rotary units mechanical transmission mathematical model. *Mathematical Modeling*, 2(41), 53—61. https://doi.org/10.31319/2519-8106.2(41)2019.185045
16. Romaniuk, O. D. (2020). Optimization of masses of shafts and bearings of mechanical transfer. *Mathematical Modeling*, 1(42), 41—47. https://doi.org/10.31319/2519-8106.1(42)2020.206940 [in Ukrainian].
17. Romaniuk, O. D. (2021). Optimization of stepped gears according to strength criteria. *Mathematical Modeling*, 1(42), 41—47. https://doi.org/10.31319/2519-8106.1(44)2021.235932 [in Ukrainian].
18. Pavlyshhe, V. T. (2013). *Fundamentals of design and calculation of machine parts*. Kyiv [in Ukrainian].
19. Pisarenko, G. S., Yakovlev, A. P., Matveev, V. V. (1988). *Handbook of Strength of Materials* (Ed. by G. S. Pisarenko). Kyiv [in Russian].

Received 02.02.2024
Revised 22.06.2024
Accepted 01.07.2024

О.Д. Романюк (https://orsid.org/000-0003-1931-5507),
Д.П. Часов (https://orsid.org/0000-0003-3830-693X),
Я.О. Романюк (https://orsid.org/0009-0001-1343-2211),
О.О. Сасов (https://orsid.org/0000-0002-8697-6324),
В.О. Бейгул (https://orsid.org/0000-0003-1350-0656)

Дніпровський державний технічний університет,
вул. Дніпробудівська, 2, Кам'янське, 51918,
Дніпропетровська область, Україна,
+380 569 56 0665, science@dstu.dp.ua

ОПТИМІЗАЦІЯ МАС ОБЕРТАЛЬНИХ ВУЗЛІВ МЕХАНІЧНИХ ПЕРЕДАЧ МАШИННОГО АГРЕГАТУ

Вступ. Необхідність стабілізації кінематичних та динамічних характеристик машинного агрегату, які мають періодичний характер зміни при усталеному режимі роботи, обумовлює пошук методів підбору махових мас для підвищення продуктивності виконавчого органу машини.

Проблематика. Стабілізацію параметрів машинного агрегату забезпечують додаткові махові маси, що призводить до збільшення ваги механізму та його інерційності. Тому доцільно проєктувати нові механічні передачі з відповідними масами обертальних вузлів, які могли б виконувати функцію махової маси.

Мета. Розроблення методу оцінки маси обертального вузла механічної передачі на етапі попереднього проєктування з використанням силових параметрів машинного агрегату.

Матеріали й методи. Використано аналітичні методи дослідження залежностей, отриманих на основі моделі машинного агрегату та обертального вузла механічної передачі з циліндричними зубчастими колесами.

Результати. Встановлено функціональну залежність маси обертального вузла механічної передачі від крутного моменту, передаточного числа та механічних характеристик матеріалів зубчастих коліс і валу. Введені коефіцієнти маси зубчастих коліс та валу дають можливість варіювати вибором матеріалу або термообробкою, що суттєво розширює діапазон пошуку оптимального варіанту та спрощує вирішення поставленої багатоваріантної задачі конструювання.

Висновки. Запропонований метод оцінки маси обертального вузла механічної передачі на етапі попереднього проєктування на основі силових параметрів машинного агрегату є перспективним, оскільки відповідні силові характеристики та механічні властивості матеріалів зубчастих коліс та валів визначені однозначно. Такий підхід дозволить підібрати відповідну масу ланок складальних одиниць, моменти інерції яких відповідатимуть маховому моменту маховика, що, у свою чергу, забезпечить необхідну стабілізацію кінематичних та динамічних характеристик машинного агрегату.