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PREDICTION OF OPERATIONAL AND EMERGENCY STATES OF ROCKET EQUIPMENT CRITICAL SYSTEMS UNDER REPEATED THERMAL AND POWER LOAD

A mathematical model for studying the thermal elastic and plastic stress-strain state and the strength of the rocket equipment systems at restarts has been proposed using the heat conductivity equations and the constitutive equations of thermal plasticity for repeated elastic-plastic strains of isotropic materials along small-curvature trajectories, the strength and low-cyclic fatigue criteria, numerical methods for solving the boundary heat conductivity problems, as well as special software.

Keywords: mathematical model, structural element, thermo-elastic-plastic strain, repeated load, strength criteria, and low-cyclic fatigue criteria.

STATEMENT OF THE PROBLEM

The creation of methods for predicting the operational and emergency states of launch equipment under restarts includes the development of calculation methods and approaches for determining the temperature and stress strain state (SSS) of critical structural elements of equipment under restarts and for estimating their residual service life. It is assumed that for estimating the residual service life of the structure it is necessary to do this for its elements undergoing the heaviest thermal and power loads under restarts. It is necessary to determine the temperature distribution in the elements and proceeding from the constitutive equations describing the repeated thermoplastic strains of material of these elements to formulate respective boundary problems, to create or to improve known methods and software, to calculate SSS, to estimate strength of elements, and to prove the reliability of results.

In [1—7], the methods for solving the thermal plasticity boundary problems of the rotation shells and bodies strained beyond the elasticity limits using numerical integration, methods of finite differences and finite elements and software [8—10], which enable determining the temperature field and elastic-plastic SSS for the rotation bodies and shells according to various theories of plasticity. However, they are not suitable for estimating the strength of specific elements of launch equipment, since the existing versions of software are not designed for numerical study of intensive thermal and power loads acing on these elements under short-term combined effect of thermal radiation, convective heat transfer, and heat flux during the first rocket launch and restarts. In addition, some

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critical elements of launch equipment have a plate-like shape. So, to calculate the SSS of elements, it is necessary to have methods that take into account the specific physical, mechanical, and geometrical factors. In this regard, for the purpose of this research, available techniques for solving thermal plasticity problems of rotation bodies and shells and related software have been improved in terms of their application to the calculation of launch equipment elements. To calculate the plate-like elements, the ANSYS licensed software package [11] granted by *Pivdenne* design office as part of the agreement on research and technical cooperation has been used.

The subject of this research is a launch pad whose dimensions, material properties, and conditions of thermal and power impact on its individual elements are provided by *Pivdenne* Design Office. For each of the pad critical elements, it is necessary to formulate the respective thermal plasticity problem and, having solved it, to conclude on the strength of this element under restarts.

GENERAL STATEMENT OF THE PROBLEM OF BODY'S SSS UNDER HEATING AND LOADING

In general, any structural element is a solid deformable body having a certain volume and limited by given surface, which at the initial time t0 has a natural stress-free state, at the initial temperature T0. Then, the body is subjected to heating and loading by the external volume forces acting on each elementary volume and by the surface forces acting on the part of body surface. The other parts of the body can be fixed.

The following assumptions are used:

1) The body is heated and loaded so slowly that the loading can be seen as a set of equilibrium states, with the body deformation not entailing any change in its temperature; hence, the loading is deemed a quasi-static process;

2) The action of external loads and uneven heating causes small deformations of the elements; the element material before the loading is isotropic; it is deformed both within and beyond the elasticity limits, with the creep strain being negligible as compared with the elastic and plastic components.

3) In the area of inelastic strain, relief is possible with appearance of secondary plastic strains, with the material having the ideal Bauschinger effect.

To solve the problem of SSS under heating and loading, it is necessary to choose a coordinate system depending on the body shape, as well as to set its geometrical parameters, thermal and mechanical properties of the material and conditions of heat exchange with the environment. Heating and loading should be divided into several stages, so that the time intervals separating the stages coincide with the time of transition from the active stress to the relief and vice versa. At the end of any stage of load, on the basis of given information it is necessary to determine the temperature distribution in the body, three components of the displacement vector at every point, six components of the strain tensor, and six components of the stress tensor, i.e. 16 unknown functions totally. To determine the temperature, the heat conduction problem should be solved for given initial and boundary conditions.

Having determined the temperature distribution in the body at an arbitrary time and set the appropriate loads and grip conditions, one can find the components of displacement vector, as well as of strain and stress tensors, which meet static, geometric, and constitutive equations and specified boundary conditions. The static equations are three equilibrium state differential equations [2, 5]; the geometric equations are six Cauchy relations between the components of the strain tensor and the displacement vector [2, 5]; and six constitutive equations form connection between the components of the stress σ_{ij} ($i, j = 1, 2, 3$) and the strain ε_{ij} ($i, j = 1, 2, 3$). tensors. The constitutive equations are the relations of the strain theory for trajectories of small curvature linearized by the method of additional stresses [1—5].

The estimation of thermoplastic SSS of the structural element is solved by iteration approximations followed by estimating its strength. If after the first load, SSS of structural element does not reach the fracture level, it is necessary to calculate (for example, on the basis of varying hysteresis loop of plastic strain or amplitude of strain) how many repeated loads this element can endure.

JUSTIFICATION OF STUDYING THE THERMAL ELASTIC AND PLASTIC STATES OF PLATE STRUCTURES USING SOLUTIONS FOR AXISYMMETRIC BODIES

Usually, the launch equipment has a sophisticated geometry and structure. This fact complicates their numerical study. However, in some cases, the SSS of structural elements can be estimated using the results obtained for the bodies of simpler geometry, for example, the problem of uneven heating in the case of the thin-walled cylinder and the long box-type structure both made of the same material. The width of the box wall is assumed to be equal to the external diameter of the cylinder, with the thickness of their walls being identical (Fig. 1). The initial and boundary conditions on the outer and inner surfaces of both bodies are considered identical. For the calculations, the reference point of the coordinate system is placed in the center of the box. The problem of thermoplastic deformation of thin-walled cylinder is solved using the axisymmetric shell theory in the formulation [6] and the ANSYS finite element software system [11] that applies also to solving the problem of SSS of box-type structure under planar strain. As a result of calculations, the temperature distributions over the thickness of the cylinder and the box-type structure, which are obtained using different methods have been found to be identical. Also, it has been established that in the intersections corresponding to the middle of box walls, there is a good agreement of the stresses having the same direction in the box and in the cylinder. This means that the method for solving the problem thermal plasticity under repeated loads can be developed and tested in the axisymmetric formulation and then be used for the plate structures.

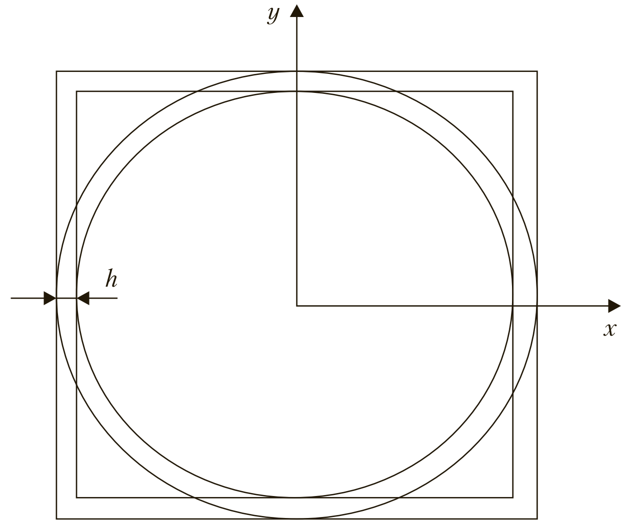

Fig. 1. Intersection of box-like and cylindrical structures

METHOD FOR STUDYING TEMPERATURE FIELDS AT RESTARTS

The launch pad is a box-type structure whose elements are made of isotropic materials. As a result of restarts, these elements undergo intensive heating and subsequent cooling, with significant strains appearing in the material of elements. These strains causes anisotropy of materials that initially are isotropic. Therefore, it is expedient to study the unsteady temperature fields in anisotropic elements.

For determining the temperature field in anisotropic elements the differential equation of heat conductivity [12] is used:

$$c\rho \frac{\partial T}{\partial t} = -div\,\vec{p},\ \vec{p} = -\lambda gradT\,, \qquad (1)$$

where T is temperature, t is time, $\vec{p}$ is heat flux vector, c is specific heat capacity of material, ρ — density of material, $\lambda = \lambda(\lambda_{ij})$ is heat conductivity coefficient.

It is assumed that in the body, there are no heat sources, and heat emission as a result of deformations can be neglected.

The differential equation (1) can be integrated at the initial and at the boundary conditions:

$$T = T_o\,,\ t = t_o\,, \qquad (2)$$

$$\vec{n}\vec{p} = \alpha(T - \theta) + \vec{n}\vec{p}. \quad (3)$$

The last condition is set on the surface limiting the body. Here, $\vec{n}$ is external normal to the surface limiting the body, α is heat exchange coefficient, θ is temperature of environment, and $\vec{p}$ is external heat flux.

The expression (3) is a condition of convective heat exchange with environment according to the Newton law, at given external heat flux. In the general case, the components of heat conductivity coefficient tensor are temperature functions. The condition (3) covers three types of boundary conditions of unsteady heat conductivity problem. At $\alpha \to \infty$, the condition (3) corresponds to given temperature on the body surface; at $\alpha = 0$, it is referred to given heat flux on the body surface. This expression can be used in the case of heating the body according to the Stefan—Boltzmann law, for the radiant heat exchange between two surfaces. In the expression (3) $\alpha = \alpha_{н} + \alpha_{сб}$, where the first additive corresponds to the heat exchange according to the Newton law, whereas the second one is related to the heating in accordance with the Stefan-Boltzmann law.

Despite the fact that the launch pad is a square box-like structure, when determining the temperature in its elements can be reduced to a non-axisymmetric heat conduction problem for the rotation body under heating conditions varying in the circumferential direction. In this case, the correlation of boundary points can be found using conformal transformation of the square to the circle. Insofar as the initially isotropic material, after repeated starts, can become an orthotropic one, the variation heat equation for cylindrical rotation body made of orthotropic material [12] was used. Solving the 3D heat conductivity problem directly using 3D finite elements is time-consuming and inefficient procedure for the rotation bodies. The use of semi-analytical finite element method can significantly increase its effectiveness [2, 4, 5]. This method reduces the original 3D problem to a series of 2D problems in the meridian intersection of the body. To this end, the solution is sought in the form of trigonometric series in the circumferential direction. In this case, the 3D heat conduction problem is reduced to 2D variation problems with respect to unknown coefficients of the series. Detailed calculations were made in [5]. Since the heat conductivity coefficients depend on temperature, the temperature problem is a nonlinear one. Its linearization is based on the method of iteration approximations.

As an example, an unsteady temperature field of long cylinder having an inner radius of 2.94 m and a thickness of 0.04 m has been studied under time-variable convective heat exchange with the environment and heat flux on its inner surface. The calculations have been made for four options of boundary conditions of heating: a) by convective heat transfer and given heat flux; b) only by given convective heat transfer; c) only by heat transfer according to the Stefan-Boltzmann law; d) by convective heat transfer according to the Newton law and heat exchange governed by the Stefan-Boltzmann law.

The analysis of calculation results has showed that the maximum temperature on the body surface corresponds to that at which the material can undergo structural transformations from isotropic to anisotropic one.

To determine the unsteady temperature fields of thin-walled structural elements in the form of laminated shells of rotation, a method for solving the heat conductivity problem at a given heat flux, under convection and radiation heating or their combination has been designed (unlike researches [6, 7] which take into account only the boundary conditions of convective heat exchange with the environment). The method of finite differences and explicit difference scheme in terms of time have been used. The method has been showed to be effective and to give a good correspondence between the results of calculations based thereon and made by ANSYS. It should be noted that this method can be used to determine approximate temperature of plate structural elements for the large-radius cylindrical shells.

METHOD FOR QUANTIFYING THERMOELASTIC-PLASTIC STRAIN STRESS STATE OF STRUCTURAL ELEMENTS UNDER REPEATED LOAD

Having obtained temperature distribution within the body simulating an arbitrary structural element, as a result of solving the heat conductivity problem, it is necessary to find 15 unknown functions describing SSS of the body at the end of any stage of load. For this purpose, as mentioned above, the system of equilibrium system equations, geometrical ratios, and constitutive equations should be used. The constitutive equations should be written using the method of additional stresses in the theory of strains along the trajectories of small curvature as the Hook's law with additional members [2—5]:

$$\sigma_{ij} = 2G\varepsilon_{ij} + \left[\left(K-2G\right)\varepsilon_0 - K\varepsilon_T\right]\delta_{ij} - \sigma_{ij}^{(d)},$$

$$\sigma_{ij}^{(d)} = 2G\sum_{k=1}^{M}\Delta_k e_{ij}^{(p)}; \quad (4)$$

$$K = \frac{E}{1-2\nu} \quad E = 2G\left(1+\nu\right);$$

$$\varepsilon_0 = \varepsilon_{ii}/3; \; \varepsilon_T = \alpha_T\left(T-T_0\right). \quad (5)$$

Where E, G are elasticity modulus and shear modulus, respectively, ν, α_T are Poisson ratio and linear thermal expansion coefficient; δ_{ij} is Kronecker's delta function; Δ_k is increase in the respective value for the k-th stage; $e_{ij}^{(p)} = \varepsilon_{ij}^{(p)}$ are plastic components of strain tensor elements, which at the M-th stage, are equal to the sum of increases in these components for M stages;

$$e_{ij}^{(p)} = \sum_{k=1}^{M}\Delta_k e_{ij}^{(p)}, \; \Delta_k e_{ij}^{(p)} = \left\langle \frac{s_j}{S} \right\rangle_k \Delta_k \Gamma_p^*,$$

$$\Gamma_p^* = \sum_{k=1}^{M}\Delta_k \Gamma_p^*; \quad (6)$$

$$S = \left(\frac{1}{2}s_{ij}s_{ij}\right)^{\frac{1}{2}}, \; s_{ij} = \sigma_{ij} - \sigma_0\delta_{ij}, \; \sigma_0 = \sigma_{ii}/3; \quad (7)$$

S is intensity of tangential strains, and Γ_p^* is intensity of accumulated plastic shear strain. In (6), the angle brackets mean average value for the respective stage. In (5), the additional stresses $\sigma_{ij}^{(d)}$ are deemed known and obtained from solving the problem at the previous stages and approximations. Their expressions contain increases in plastic strain components $\Delta_k e_{ij}^{(p)}$, which should be defined more accurately in the iteration approximations. The increase $\Delta_k e_{ij}^{(p)}$ is calculated assuming the dependence

$$S = F\left(\Gamma, T\right) \quad (8)$$

between the intensity of tangential strains S (7), intensity of shear strains Γ

$$\Gamma = \left(\frac{1}{2}e_{ij}e_{ij}\right)^{\frac{1}{2}}, \; e_{ij} = \varepsilon_{ij} - \varepsilon_0\delta_{ij} \quad (9)$$

and temperature T. For more accurate definition of dependence (8) $\sigma \sim \varepsilon$ diagrams (σ is stress, ε is elongation of the sample) obtained from experiments on stretching the cylindrical samples at various fixed values of temperature, which were made at loading rates not affecting the shape of $\sigma \sim \varepsilon$. diagrams. Transition from σ and ε to S and Γ is made according to known formulas [1—5]. For interim temperature T, the dependence (8) is found by linear interpolation. When the secondary plastic strains appear, the dependence

$$S = F_1\left(\Gamma, \Gamma_p^{(1)}, T\right) \quad (10)$$

is built using (8), intensity of accumulated plastic shear strains at the moment of upload $\Gamma_p^{(1)}$, and respective value of $S^{(1)} = F\left(\Gamma_1, T\right)$ assuming that $\Gamma_1 = \Gamma_p^{(1)} + + S^{(1)}/2G$. For uploading in the area of secondary plastic strains and for the next (repeated) load the dependence

$$S = F_2\left(\Gamma, \Gamma_p^{(2)}, T\right) \quad (11)$$

is built using (8), intensities of accumulated secondary plastic shear strains $\Gamma_p^{(2)}$ and respective values of $S^{(2)} = F_1$ (Γ_2, $\Gamma_p^{(1)}$, T). The dependences (10) and (11) are built using conditions for the ideal Bauschinger effect [13]

$$S^{(1)} + S_T^{(1)} = S^{(2)} + S_T^{(2)} = 2S_T,$$

where S_T, $S_T^{(1)}$, $S_T^{(2)}$ are intensities of tangential stresses corresponding to yield limits of material in (8), (10), and (11). for further changes in the load direction are built [14].

The constitutive equations (4), the balance equations, and the Cauchy relations form a system of

15 equations to be solved under the given boundary conditions. The solution system is formulated in terms of shifts, stresses or in a mixed form with respective boundary conditions [2, 5]. In each of these cases, a nonlinear boundary problem is obtained. Its solution at each stage of load is found by iteration approximations. At the first phase of the load, the problem of thermal elasticity is solved, while at the next stages, one deals with the problem of thermal plasticity to solve which, during each approximation, it is necessary to use the SSS components obtained at the previous stage and in the previous approximation. This common approach to determining SSS of the body simulating a particular structural element is realized using different coordinate systems, depending on the body shape and methods used for solving systems of differential equations.

The methods used for massive rotating bodies are described in detail in [2, 4, 5], while the software for solving thermal plasticity boundary problems in variation formulation using the finite element method is is given in [8, 9]. For the shell structures of arbitrary shape, an algorithm for solving this problem in the variation formulation using the finite difference method is proposed in [1]. For the thin shells, within the Kirchhoff-Love theory [15], in [1—3, 6] there are detailed descriptions of the methods for solving thermal plasticity boundary problems using the orthogonalization approach [16] for which the relevant software has been developed [8, 10].

In this research, the above mentioned methods and the appropriate software have been extended to the repeated load and to the assessment the strength of structural elements. When assessing the strength, it is assumed that for repeated stress of the body, the maximum temperature and power factors remain fixed. In this case, the deformation is characterized by varying width of the plastic hysteresis loop and by plastic strain accumulated.

In the absence of creep, there are two cases of destruction, *the quasi-static fracture* and *the fatigue distress* [17]. The fatigue distress is accompanied by fatigue cracks and small plastic strains. The quasi-static fracture is caused by the accumulation of plastic strains corresponding to single static load that causes failure of material. This kind of fracture is typical for the materials that are cyclically stable and prone to the accumulation of plastic strains. The Sdobyrev criteria [18] are used for the assessment of quasi-static fracture, whereas to estimate the low-cycle fatigue it is necessary to apply the Coffin-Manson-type criteria relating the amplitude of total or plastic strain with the number of cycles before failure.

For example, the SSS of thin cylindrical shell was considered under the action of axial force, internal pressure, and temperature, which increase and, having reached their maximum values, decrease to the zero load and the initial temperature. In the proposed method, SSS of the shell was determined under three-time repeated load. The effectiveness of iterations and the accuracy of results have been confirmed by comparing the exact solution of this problem with the solution obtained using the ANSYS complex.

STUDY OF SSS AND ESTIMATION OF RESIDUAL SERVICE LIFE OF PLATE ELEMENTS OF LAUNCHING EQUIPMENT UNDER REPEATED NON-ISOTHERMAL LOAD

Let determine SSS and estimate the residual service life of plate element having a constant thickness used for the protection of electronic equipment during the rocket launch. Let do this with the help of ANSYS software.

During the rocket launch the plate undergoes an intense thermal mechanical load. The analysis of input data has showed that the mechanical load on the plate can be neglected as compared with the thermal one. Therefore, hereafter, the research deals with thermoelastic deformation of the plate under cyclical thermal load.

The geometry of a plate quarter with a grid consisting of finite number of elements is given in Fig. 2. At the crossing $x = 0$ and $y = 0$ the symmetry conditions are set. Beyond the contour, the plate is deemed thermally insulated. On the inner surface $z = 0$, the conditions of convective heat exchange with environment having a temperature of T_{cp} = 308 K. are established. Heat exchange coefficient on this surface is α = 35 W/m^2 · K. Initial temperature of the plate is T_0 = 308 K. The load cycle consists of heating and cooling processes. At the first stage, the outer surface z = = 0.03 m is heating during 13.2 s in accordance with boundary condition (3), where $\alpha = \alpha_н$. Temperature of external environment, heat exchange coefficient, and specific radiant heat flux vary with time reaching their maximums at 2.8 s $\leq t \leq$ 3.2s. Having been heated the plate is cooling down to the initial temperature T_0 during 6000 s, with heat exchange conditions on the surface z = 0.03 m being identical to those on the surface z = 0. The plate is made of 10ХСНД steel. This material is deemed to harden linearly with ideal Bauschinger effect.

The calculations have showed that while heating and cooling the plate, the temperature T and stress σ_{ij} distributions vary insignificantly at the crossings parallel to Oxy plane. The shear stresses can be neglected as compared with the normal ones, while stresses σ_{xx} and σ_{yy} differ slightly from each other and exceed stress σ_{zz} by an order of magnitude. It has been established that being heated during 2 s the plate warms up a little bit, whereas at t = 2.8 s significant temperature gradients appear and cause compressive stresses near the plate surface. As the heating rate decreases, temperature is distributed uniformly across the plate thickness, with the stresses reversing their sign due to a plastic deformation in the area of maximum temperature gradients, neat the plate surface.

Some results of calculations are showed in Fig.3. In Figs 3 *a* and *b*, one can see temperature T dependence with time at the point P_0 with coordinates $x = y = 0$, z = 0.03 m. Fig. 3, *a* shows temperature dependence for 5 cycles of heating and cooling; Fig. 3, *b* illustrates change in temperature during heating for 1st and 2nd cycles. One can see that after the cooling, temperature reaches its initial value T_0, with the heating for different cycles varying insignificantly. In Fig. 3,

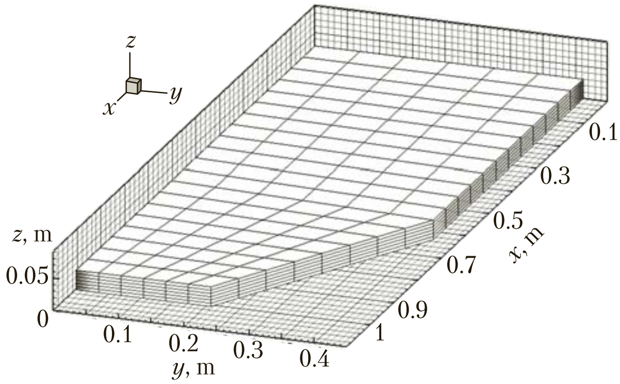

Fig. 2. Geometry and finite-element grid of the plate

c, σ_{xx} stress distribution is given at the point P_0 during the heating for 1st, 2nd, and 5th cycles. Figs. 3, *b* and *c* show that stresses σ_{xx} reach their maximums at the moment of maximum warm-up of the plate. Having compared stresses at the end of heating of the 1st cycle and at the beginning of the 2nd and 5th cycles, one can see that during the cooling the stresses do not vary materially. Fig. 3, *d* features dependence $sign(I_3(D_\sigma))S \sim \Gamma$ at the point P_0 for 5 cycles of load (intensity of shear stresses S multiplied by sign of the 3rd stress deviator $I_3(D_\sigma)$ for determining the sign of load). As one can see from Fig. 3, *d*, the hysteresis loop is stabilized at the 5th load cycle, with the amplitude of total strain being $\Delta\varepsilon = 2/\sqrt{3}\,\Delta\Gamma \sim 0.34$ %. The residual life of this structure under repeated thermal load is estimated using the Coffin-Manson type fatigue criterion [19]:

$$\Delta\varepsilon\, N^m = A\left(D^n + \frac{\sigma_B}{E}\right), \qquad (12)$$

where $\Delta\varepsilon$ is amplitude of total strain; N is number of cycles before failure; D, A, n, m are constants depending on material properties. The properties of material in (12) correspond to maximum temperature of the cycle and are taken accordingly to the lower estimate of yielding steel durability [2]. The number of cycles before failure of the structure under consideration has been established to be $N \geq 3000$. To estimate the residual service life of this structure more accurately is impossible unless reliable data on cyclical plastic deforma-

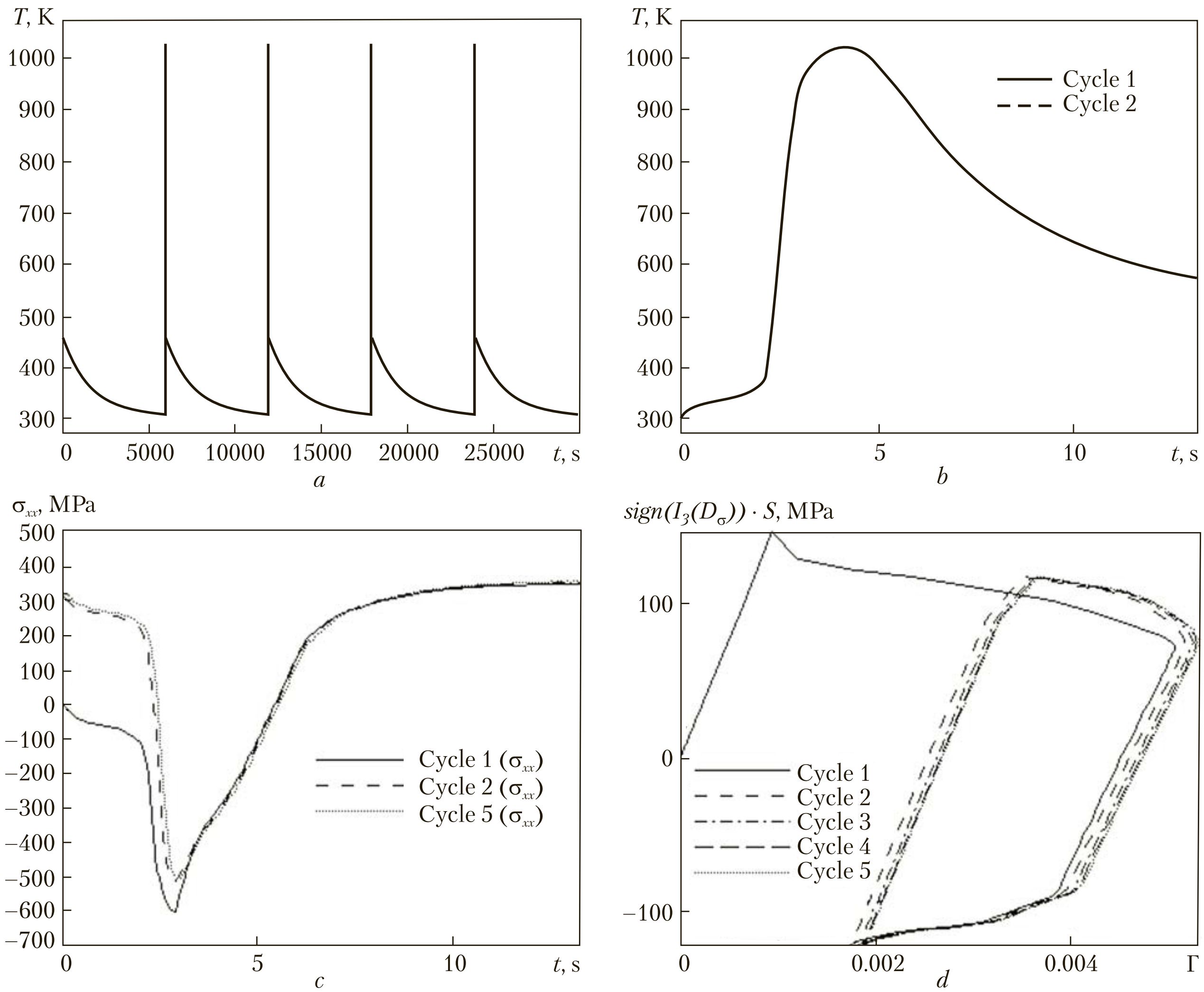

Fig. 3. Temperature distribution with time (*a, b*) and SSS (*c, d*) of the plate under repeated heating

tion and thermal fatigue strength of the material used are available. The detailed calculations are given in [21].

CONCLUSIONS

1. A mathematical model has been proposed for studying thermoelastic plastic SSS and strength of launch pad elements. The model enables estimating the residual service life of the structure.

2. A general statement of the problem has been formulated. A method for estimating the strength and residual service life of the structure consisting of many elements of various geometry under repeated power and thermal loading has been proposed. The estimate of strength and residual service life is based on computation of temperature and SSS of the most stressed members. For this purpose, the existing methods and software for computing the temperature fields of thin- and thick-walled rotation bodies have been improved, including by solving non-steady heat conductivity problem with actual modes of heating taken into account due to formulating respective boundary conditions.

3. The existing methods and software for solving the rotation body thermoelasticity problem have been generalized by developing an effective

algorithm for specifying correlations between the second invariants of stress and strain deviators under variable load with secondary plastic deformations and ideal Bauschinger effect taken into consideration.

4. The results of the study of launch pad plate elements have been given as an example of application of the method for estimating the strength and residual service life of the structure. The results of computation of temperature field and SSS have showed for after the fifth launch the SSS components do not change and the structure endures further launches.

5. The number of launches before failure has been estimated. This estimate is fair under fixed thermoelastic load on launching equipment provided the mechanical properties of material are stable in the course of operation.

It should be noted that the obtained results are approximate since the calculations are made on the basis of input data on material properties taken from available literature, which roughly correspond to the actual properties of material. The strength of launch pad elements can be accurately estimated using this method provided the diagrams of material cyclic deformations obtained after each launch of the booster are available.

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МЕТОДИКА ПРОГНОЗУВАННЯ ЕКСПЛУАТАЦІЙНОГО І ГРАНИЧНОГО СТАНУ ВІДПОВІДАЛЬНИХ СИСТЕМ РАКЕТНОЇ ТЕХНІКИ ПРИ ПОВТОРНИХ ТЕРМОСИЛОВИХ НАВАНТАЖЕННЯХ

Запропоновано математичну модель для дослідження термопружнопластичного напружено-деформованого стану та міцності систем ракетної техніки при повторних пусках, яка дозволяє оцінити ресурс конструкції. Використано рівняння теплопровідності та визначальні рівняння термопластичності для процесів повторного пружнопластичного деформування ізотропних матеріалів вздовж траєкторій малої кривизни, критерії міцності та малоциклової втоми, чисельні методи розв'язання крайових задач теплопровідності та термопластичності, а також відповідні комп'ютерні програми.

Ключові слова: математична модель, елемент конструкції, процес термопружнопластичного деформування, повторне навантаження, критерій міцності, критерій малоциклової втоми.

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МЕТОДИКА ПРОГНОЗИРОВАНИЯ ЭКСПЛУАТАЦИОННОГО И ПРЕДЕЛЬНОГО СОСТОЯНИЯ ОТВЕТСТВЕННЫХ СИСТЕМ РАКЕТНОЙ ТЕХНИКИ ПРИ ПОВТОРНЫХ ТЕРМОСИЛОВЫХ НАГРУЖЕНИЯХ

Предложена математическая модель для исследования термоупруго-пластического напряженно деформированного состояния и прочности систем ракетной техники при повторных пусках, позволяющая оценить ресурс конструкции. Использованы уравнение теплопроводности и определяющие уравнения термопластичности для процессов повторного упруго пластического деформирования изотропных материалов по траекториям малой кривизны, критерии прочности и малоцикловой усталости, численные методы решения краевых задач теплопроводности и термопластичности и соответствующие компьютерные программы.

Ключевые слова: математическая модель, элемент конструкции, процесс термоупруго-пластического деформирования, повторное нагружение, критерий прочности, критерий малоцикловой усталости.

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